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Non-uniform mass transfer or wall enthalpy into a compressible flow over yawed cylinder

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Abstract

An analysis is performed to obtain the non-similar solution of a steady compressible laminar boundary layer flow over a yawed infinite circular cylinder with non-uniform slot injection (or suction) and non-uniform wall enthalpy. The difficulties arising at the starting point of the streamwise coordinate, at the edges of the slot and at the point of separation are overcome by applying the method of quasilinear implicit finite-difference scheme. It is observed that the separation can be delayed by non-uniform slot suction and also by moving the slot downstream but the effect of non-uniform slot injection is just the reverse. An increase in Mach number and total enthalpy at wall causes the separation to occur earlier while cooling delays it. The non-uniform total enthalpy at the wall (i.e., the cooling or heating of the wall in a slot) has very little effect on the skin friction and hence on the point of separation.

1. Introduction

Studies relating to compressible boundary layer flow over a yawed infinite circular cylinder have come into prominence due to the recent development of modern aircraft, guided missiles, etc. In particular, the yawed infinite cylinder simulates approximately the leading edge of a swept-back wing or a body of high fineness ratio at an angle of attack and also allows a basic simplification of the complicated three-dimensional compressible boundary layer equations. Not surprisingly this problem has attracted the attention of many investigators [1–7]. Furthermore, detailed survey of the literature on flow past a yawed infinite cylinder has been made by Dewey and Gross [8] but in the past studies, the exact location of the point of separation was not reached. Using advanced numerical techniques, the accurate prediction of the point of separation may be helpful in reducing the energy losses due to the formation of boundary layer and its separation. The nature of steady three-dimensional laminar boundary layer separation may be characterized by two possible modes, namely, singular separation and ordinary separation as has been pointed out by Maskell [9]. For singular separation, both components of the wall shear vanish simultaneously and for ordinary separation, only one component of the wall shear vanishes. Ordinary separation appears to be the dominant form of separation on most three-dimensional bodies. Excellent reviews of the phenomenon of separation of boundary layer flows have been given by Cebeci et al. [10] and Smith [11].

Mass transfer from a wall slot (i.e., mass injection or suction occurs in a small porous section of the body surface and there is no mass transfer in the remaining part of the body surface) into the boundary layer strongly influences the development of a boundary layer along a surface and in particular can prevent or at least delay separation of the viscous region. Different studies [12–15] show the effect of slot injection (suction) into laminar compressible boundary layer over a flat plate by taking the interaction between the boundary layer and oncoming stream. Uniform mass transfer in a slot causes finite discontinuity at the leading and trailing edges of the slot. The discontinuities can be avoided by choosing a non-uniform mass transfer/wall temperature distribution in the slot [16,17]. In a

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Nomenclature

A	dimensionless mass transfer parameter
C_1, C_2	constants
$C_f, \bar{C}_f$	skin friction coefficients in the x- and z-directions, respectively
C_p	specific heat at constant pressure
E	dimensionless dissipation parameter
f	dimensionless stream function
$F\ (= f_\eta)$	dimensionless velocity component in the x-direction
G	dimensionless total enthalpy
h	static enthalpy
H	total enthalpy
k	thermal conductivity
M_∞	free stream Mach number
N	$(\rho\mu/\rho_e\mu_e)$, Chapman–Rubesin function
R	characteristic radius of the body
Re	Reynolds number
S	dimensionless velocity component in the z-direction
St	Stanton number
T	temperature
u, v, w	dimensional velocity components in the x-, y- and z-directions, respectively
U_∞	resultant free stream velocity
x, z	dimensional chordwise and spanwise distances, respectively
$\bar{x}$	x/R, dimensionless chordwise distance
y	distance normal to the surface

Greek symbols

β	pressure gradient parameter
γ	ratio of specific heats
η, ξ	transformed coordinates
θ	yaw angle
μ	dynamic viscosity
ν	kinematic viscosity
ρ	density
ψ	dimensional stream function

Subscripts

∞	conditions in the freestream
e, w	denote conditions at the edge of the boundary layer and on the surface, respectively
$\bar{x}, \xi, \eta$	denote partial derivatives with respect to these variables

recent investigation, Roy and Nath [17] have studied the effects of non-uniform slot injection/suction combinations and non-uniform total enthalpy at the wall into a steady non-similar compressible boundary layer flow over two-dimensional and axisymmetric bodies.

The purpose of this investigation is to analyze the steady non-similar compressible flow over a yawed infinite circular cylinder using non-uniform slot injection or suction (i.e., mass transfer occurs in a small porous section of the body surface and the remaining part of the body surface is solid) and non-uniform total enthalpy at the wall (wall cooling or heating takes place in a slot). The present analysis may be useful in understanding many boundary layer problems of practical importance as would arise, for example, in cooling gas turbine blades, suppressing recirculating bubbles and controlling transition and/or separation of the boundary layer over airplane control surfaces. The non-similar solutions have been obtained starting from the origin of the streamwise coordinate to the point of separation (zero skin friction in the streamwise direction) using quasi-linearization technique with an implicit finite-difference scheme.

It may be noted that the discontinuities at the leading and trailing edges of the slot have been avoided following [16,17]. Thus, the present analysis differs from those in [12–15] with finite discontinuities.

2. Analysis

Consider the boundary layer flow over a yawed infinite circular cylinder placed in a uniform compressible flow of velocity U_∞ as shown in Fig. 1. The blowing rate is assumed to be small and it does not affect the inviscid flow at the edge of the boundary layer. It is also assumed that the injected fluid possesses the same physical properties as the boundary layer fluid and has a static temperature equal to the wall temperature. The Prandtl

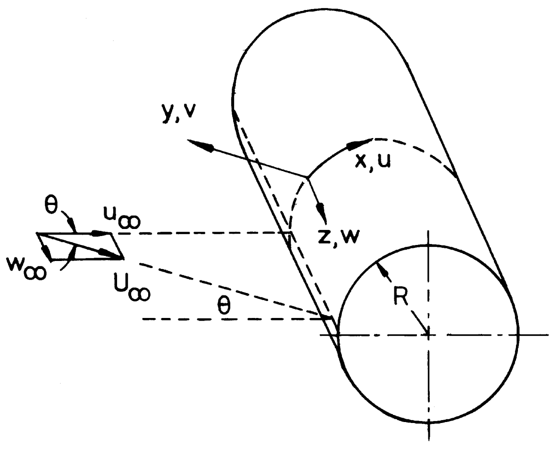

Fig. 1. Flow model and coordinate system.

number Pr is assumed to be constant as its variation across the boundary layer is negligible for most of the atmospheric problems [18]. Under the foregoing assumptions, transformed boundary layer equations can be expressed in a non-dimensional form as [5,7,8,19]

$$(NF_\eta)_\eta + fF_\eta + \beta(\xi)\{(\rho_e/\rho) - F^2\} = 2\xi(FF_\xi - F_\eta f_\xi), \tag{1}$$

$$(NS_\eta)_\eta + fS_\eta = 2\xi(FS_\xi - S_\eta f_\xi), \tag{2}$$

$$(NG_\eta)_\eta + Pr\, fG_\eta - 2(1-Pr)E\Big[NFF_\eta(u_e/u_\infty)^2\cos^2\theta + NSS_\eta\sin^2\theta\Big]_\eta = 2\,Pr\,\xi(FG_\xi - G_\eta f_\xi), \tag{3}$$

where

$$\begin{aligned}
&\xi = \int_0^x \rho_e\mu_e u_e\,dx, \quad \eta = u_e(2\xi)^{-1/2}\int_0^y \rho\,dy,\\
&\psi(x,y) = (2\xi)^{1/2}f(\xi,\eta),\\
&\rho u = (\partial\psi/\partial y),\\
&\rho v = -(\partial\psi/\partial x),\\
&u = u_e f_\eta(\xi,\eta) = u_e F(\xi,\eta), \quad w = w_e S(\xi,\eta),\\
&H = H_e G(\xi,\eta), \quad H_e = H_\infty = \text{constant},\\
&H = h + 2^{-1}(u^2+w^2) = C_p T + 2^{-1}(u^2+w^2),\\
&N = \rho\mu/(\rho_e\mu_e), \quad \beta(\xi) = (2\xi/u_e)(du_e/d\xi),\\
&E = U_\infty^2/(2H_e) = (\gamma-1)M_\infty^2/[2+(\gamma-1)M_\infty^2].
\end{aligned} \tag{4}$$

The boundary conditions imposed on the set of equations (1)–(3) are

$$\begin{aligned}
&F(\xi,0) = 0,\\
&S(\xi,0) = 0, \quad G(\xi,0) = G_w P(\bar{x}),\\
&F(\xi,\infty) = 1,\\
&S(\xi,\infty) = 1, \quad G(\xi,\infty) = 1,
\end{aligned} \tag{5}$$

where $G_w P(\bar{x})$ is the total enthalpy distribution to be prescribed at the wall and

$$f = \int_0^\eta F\,d\eta + f_w, \quad f_w = -(2\xi)^{-1/2}\int_0^x (\rho v)_w\,dx. \tag{6}$$

It may be noted that the set of equations (1)–(3) reduces to that of two-dimensional case for $\theta = 0$. Hence, Eq. (2) becomes redundant as the velocity component in the z-direction $w = 0$ (i.e., $S = 0$) for $\theta = 0$.

The potential flow velocity for a yawed infinite circular cylinder of radius R in subsonic flow is given by [20]

$$\begin{aligned}
&u_e(\bar{x}) = u_\infty(C_1\sin\bar{x} + C_2\sin 3\bar{x}),\\
&u_\infty = U_\infty\cos\theta,\\
&w_e = w_\infty = U_\infty\sin\theta = \text{constant},
\end{aligned} \tag{7}$$

where $C_1 = 2(1 + M_\infty^2/3)$, $C_2 = -M_\infty^2/2$. It may be remarked that in Eq. (7), which is valid for subsonic flow ($M_\infty \leqslant 0.4$), we have retained the terms up to order M_∞^2 only. If the terms up to the next order, i.e., up to order M_∞^4 are included in the series (7), the flow on the surface of cylinder at $\bar{x} = \pm\pi/2$ becomes locally supersonic [21] at $M_\infty \approx 0.404$. For $0.404 < M_\infty < 0.45$, there are regions just above and just below the cylinder in which the flow is supersonic and there is an apparently smooth transition from subsonic flow elsewhere. For $M_\infty > 0.45$ the series (7) appears to diverge [21]. Experimentally it has been found that local shock waves appear for about $M_\infty = 0.45$. Consequently, as a first step, we have investigated only the subsonic flow ($M_\infty \leqslant 0.4$).

Using Eqs. (4), (6) and (7), the expressions for ξ, β and f_w can be written as

$$\xi = R\rho_e\mu_e u_\infty P_3, \quad \beta = 2\cos\bar{x}P_4P_5P_2^{-1}P_6^{-2}, \tag{8}$$

$$f_w = \begin{cases} 0 & \text{for } \bar{x} \leqslant \bar{x}_0,\\ A(\cos\theta)^{-1/2}P_3^{-1/2}C(\bar{x},\bar{x}_0) & \text{for } \bar{x}_0 \leqslant \bar{x} \leqslant \bar{x}_0^*,\\ A(\cos\theta)^{-1/2}P_3^{-1/2}C(\bar{x}_0^*,\bar{x}_0) & \text{for } \bar{x} \geqslant \bar{x}_0^*, \end{cases} \tag{9}$$

where the function $C(\bar{x},\bar{x}_0) = 1 - \cos\{w^*(\bar{x}-\bar{x}_0)\}$ and

$$\begin{aligned}
&P_1 = 1 - \cos\bar{x}, \quad P_2 = 1 + \cos\bar{x},\\
&P_3 = C_1P_1 + C_2(1 - \cos 3\bar{x})/3,\\
&P_4 = C_1 + 3C_2\Big(4\cos^2\bar{x} - 3\Big),\\
&P_5 = C_1 + C_2(1 + 4\cos\bar{x}P_2)/3,\\
&P_6 = C_1 + C_2\Big(3 - 4\sin^2\bar{x}\Big).
\end{aligned}$$

Here $(\rho v)_w$ is taken as

$$(\rho v)_w = -(\rho_e U_\infty)(Re/2)^{-1/2}Aw^*\sin\{w^*(\bar{x}-\bar{x}_0)\},$$
$$\bar{x}_0 \leqslant \bar{x} \leqslant \bar{x}_0^* = 0, \ \ \bar{x} \leqslant \bar{x}_0 \ \text{ or } \ \bar{x} \geqslant \bar{x}_0^*,$$

where $Re = (U_\infty R/\nu_e)$ and, w^* and $\bar{x}_0$ are the two free parameters which, respectively, determine the slot length and slot location. The function $(\rho v)_w$ is continuous for all values of $\bar{x}$ and it has a non-zero value only in the interval $[\bar{x}_0, \bar{x}_0^*]$. The reason for taking such type of function is that it allows the mass transfer to change slowly in the neighbourhood of the leading and trailing edges of the slot. The parameter $A > 0$ or < 0 according to whether there is a suction or injection. Since the flow considered here is a subsonic one, it is reasonable to take the fluid medium as one which has constant gas properties. Accordingly, we have $\rho \propto h^{-1}$, $\mu \propto h$, $Pr =$ constant and

$$\rho_e/\rho = \Big[G - E\{(u_e/u_\infty)^2\cos^2\theta F^2 + \sin^2\theta S^2\}\Big]\Big/\Big[1 - E\{(u_e/u_\infty)^2\cos^2\theta + \sin^2\theta\}\Big]. \tag{10}$$

Eqs. (1)–(3) can be expressed in terms of $\bar{x}$ using relation (8) between ξ and $\bar{x}$ as

$$\xi(\partial/\partial\xi) = B(\bar{x})(\partial/\partial\bar{x}), \tag{11}$$

where $B(\bar{x}) = P_3 P_6^{-1}/(\sin\bar{x})$. Substituting Eqs. (10) and (11) in Eqs. (1)–(3), we obtain

$$F_{\eta\eta} + fF_\eta + \beta_1\left[G - F^2 + E_1\left(F^2 - S^2\right)\right] = 2B(\bar{x})(FF_{\bar{x}} - F_\eta f_{\bar{x}}), \tag{12}$$

$$S_{\eta\eta} + fS_\eta = 2B(\bar{x})(FS_{\bar{x}} - S_\eta f_{\bar{x}}), \tag{13}$$

$$G_{\eta\eta} + Pr\,fG_\eta - 2(1-Pr)\left[E_2FF_{\eta\eta} + E_2F_\eta^2 + E_1SS_{\eta\eta} + E_1S_\eta^2\right] = 2PrB(\bar{x})(FG_{\bar{x}} - G_\eta f_{\bar{x}}), \tag{14}$$

where $E_1 = E\sin^2\theta$, $E_2 = E(u_e/u_\infty)^2\cos^2\theta$ and $\beta_1 = \beta/[1 - E_1 - E_2]$. The boundary conditions become

$$\begin{aligned} &F(\bar{x},0) = 0, \quad S(\bar{x},0) = 0, \quad G(\bar{x},0) = G_w P(\bar{x}), \\ &F(\bar{x},\infty) = 1, \quad S(\bar{x},\infty) = 1, \quad G(\bar{x},\infty) = 1, \end{aligned} \tag{15}$$

where $f = \int_0^\eta F\,d\eta + f_w$ and f_w is given by Eq. (9). Here the total enthalpy distribution at the wall along the dimensionless streamwise coordinate ($\bar{x}$) is chosen as $G_w P(\bar{x})$. G_w is a constant and the function $P(\bar{x})$, associated with the non-uniformity of the total enthalpy at the wall, is given by

$$P(\bar{x}) = \begin{cases} 1 + \varepsilon\sin\{w^*(\bar{x} - \bar{x}_0)\}, & \bar{x}_0 \leqslant \bar{x} \leqslant \bar{x}_0^*, \\ 1, & \bar{x} \leqslant \bar{x}_0 \text{ or } \bar{x} \geqslant \bar{x}_0^*, \end{cases}$$

where ε is a small real number. Further, $P(\bar{x})$ is a continuous function with a small perturbation in the interval $[\bar{x}_0, \bar{x}_0^*]$ over the constant value 1 and it gives the variation of the total enthalpy at the wall only in the interval $[\bar{x}_0, \bar{x}_0^*]$ while the remaining part of the body surface maintains the constant value of the total enthalpy. The sudden rise or fall of the total enthalpy at the edges of the slot can cause numerical difficulties in the solution of the energy equation but the use of this type of function allows the total enthalpy at the wall to change slowly in the neighbourhood of the leading and trailing edges of the slot. Similar type of function has been considered in [16,17] for the variation of the wall temperature distribution along streamwise direction.

The skin friction coefficients in x- and z-directions (i.e., in the chordwise and spanwise directions) can be expressed in the form

$$C_f(Re)^{1/2} = 2^{1/2}(\cos\theta)^{3/2}\sin\bar{x}P_2^{1/2}P_5^{-1/2}P_6^2(F_\eta)_w \tag{16}$$

and

$$\bar{C}_f(Re)^{1/2} = 2^{1/2}(\cos\theta)^{1/2}\sin\theta P_2^{1/2}P_5^{-1/2}P_6(S_\eta)_w. \tag{17}$$

Similarly, the heat transfer coefficient in terms of Stanton number is defined by

$$St(Re)^{1/2} = 2^{-1/2}[Pr(1 - G_w)]^{-1} \times (\cos\theta)^{1/2}P_2^{1/2}P_5^{-1/2}P_6(G_\eta)_w, \tag{18}$$

where

$$C_f = 2[\mu(\partial u/\partial y)]_w/\rho_e U_\infty^2,$$

$$\bar{C}_f = 2[\mu(\partial w/\partial y)]_w/\rho_e U_\infty^2$$

and

$$St = (k/C_p)(\partial H/\partial y)_w/[\rho_e(H_e - H_w)U_\infty].$$

Thus, it is clear from Eqs. (16)–(18) that $(F_\eta)_w$, $(S_\eta)_w$ and $(G_\eta)_w$ are the crucial parameters which characterize skin friction and heat transfer of the fluid flow.

3. Results and discussion

Eqs. (12)–(14) with boundary conditions (15) have been solved numerically using an implicit finite-difference scheme in combination with the quasilinearization technique [22]. The non-linear coupled partial differential equations (12)–(14) were first linearized using quasilinearization method [22]. The resulting linear partial differential equations were expressed in difference form. The equations were then reduced to a system of linear algebraic equations with a block tridiagonal structure which is solved using Varga's algorithm [23]. The stepsize in the η-direction has been chosen as $\Delta\eta = 0.01$ throughout the computation. In the $\bar{x}$-direction, $\Delta\bar{x} = 0.01$ has been used for small values of $\bar{x}$ ($\leqslant 0.50$), then it has been decreased to $\Delta\bar{x} = 0.001$. This value of $\Delta\bar{x}$ has been used for $\bar{x} \leqslant 1.25$, thereafter the stepsize has been reduced further, ultimately choosing a value $\Delta\bar{x} = 0.0001$ in the neighbourhood of the point of zero skin friction. This has been done because the convergence becomes slower when the point of vanishing skin friction in chordwise direction is approached. The choice of step sizes has been found to be optimum since further reduction does not alter the results up to the fourth decimal place.

Computations have been carried out on a OSFALPHA digital computer system for various values of A ($-0.50 \leqslant A \leqslant 0.50$), M_∞ ($0.2 \leqslant M_\infty \leqslant 0.4$), $\bar{x}_0$ ($0.50 \leqslant \bar{x}_0 \leqslant 1.25$) and G_w ($0.2 \leqslant G_w \leqslant 0.6$). In all numerical computations Pr has been taken as 0.72. The edge of the boundary layer η_∞ is taken between 4 and 6 depending on the values of parameters. One sample calculation, for example, for $M_\infty = 0.4$, $G_w = 0.6$, $A = 0.25$, $\bar{x}_0 = 1.0$ and $w^* = 2\pi$, takes approximately 1 min and 20 s CPU time in the above-mentioned computer system.

Solutions have been obtained for the non-similar incompressible flow cases by substituting $Pr = 1$ and $M_\infty = A = \theta = G_w = 0$ $[G = (T - T_w)/(T_\infty - T_w)]$ to compare the skin friction and heat transfer parameters ($(F_\eta)_w, (S_\eta)_w = (G_\eta)_w$) with those of the differential-difference method [24] and finite-difference method [25]. Also the heat transfer result ($(G_\eta)_w$) has been compared with the experimental results [26]. The results are found to be in good agreement. The results, corresponding to

$\xi = \bar{x} = 0$ in the present non-similar case, have been compared with the self-similar results obtained by Dewey and Gross [8], and found them in excellent agreement (they differ only in the fourth decimal place). We have also compared our results for zero yaw angle (i.e., for $\theta = 0$) with the steady state results of Vasantha and Nath [27] who studied the unsteady non-similar compressible boundary layer flow over a cylinder without mass transfer. The results are found to be in excellent agreement. Comparison of the results for zero yaw angle (i.e., for $\theta = 0$) is also made with the results of Roy and Nath [17] who studied recently the effect of non-uniform injection/suction combinations in a slot on a steady non-similar compressible boundary layer flow over a cylinder. The results are found to be in excellent agreement with the present results. Some of the comparisons are shown in Figs. 2 and 3 and in Table 1.

3.1. Case I: non-uniform slot injection (or suction)

Figs. 4 and 5 show the effects of non-uniform slot injection (or suction) parameter ($A < 0$ or $A > 0$) and $\bar{x}_0$ (which fixes the slot location) on the skin friction and heat transfer parameters ($(F_\eta)_w$, $(S_\eta)_w$, $(G_\eta)_w$). In the case of slot suction, the skin friction and heat transfer parameters ($(F_\eta)_w$, $(S_\eta)_w$, $(G_\eta)_w$) increase as the slot starts and attain their maximum values before the trailing edge of the slot. Finally, $(F_\eta)_w$, $(S_\eta)_w$ and $(G_\eta)_w$ decrease from their maximum values and $(F_\eta)_w$ reaches zero but $(S_\eta)_w$ and $(G_\eta)_w$ remain finite (Fig. 4). This implies (as mentioned earlier) that ordinary separation occurs at this point. For the problem under consideration, singular separation does not occur (i.e., for no value of $\bar{x}$, $(F_\eta)_w = (S_\eta)_w = 0$ simultaneously). The above results hold good whatever may be the values of the mass transfer parameter A. Hence, in subsequent discussion, for the sake of convenience, we have used the word separation to denote ordinary separation. The results indicate that the effect of slot suction is to move the

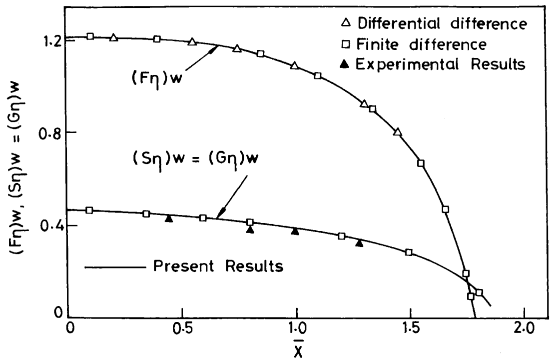

Fig. 2. Comparison of skin friction and heat transfer parameters for $Pr = 1$, $M_\infty = G_w = A = 0$.

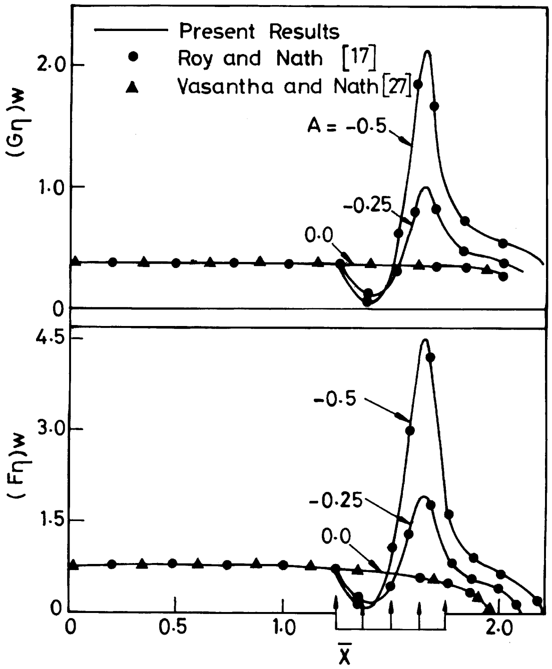

Fig. 3. Comparison of skin friction and heat transfer parameter for $M_\infty = 0.4$, $G_w = 0.2$, $w^* = 4\pi$, $\bar{x}_0 = 1.25$ and $\theta = 0$.

point of separation downstream, i.e., it delays the separation, but the injection through a slot on the body surface has the reverse effect as shown in Fig. 5. It is noted (Fig. 4) that if we move the location of the slot downstream, the point of separation also moves downstream (i.e., it delays the separation). Thus, separation can be delayed by slot suction and also by moving the slot downstream. To be more specific, for $G_w = 0.2$, $M_\infty = 0.4$ (Fig. 4), the point of separation moves downstream approximately by 15% as the rate of suction ($A > 0$) increases from 0 to 0.5.

Effects of the total enthalpy at the wall (G_w) and the freestream Mach number (M_∞) on the skin friction and heat transfer parameters ($(F_\eta)_w$, $(S_\eta)_w$, $(G_\eta)_w$) are shown in Fig. 6. For a fixed M_∞, the effect of the increase in total enthalpy at the wall (G_w) is to enhance the skin friction parameters ($(F_\eta)_w$, $(S_\eta)_w$) up to a certain region of streamwise direction and also to move the separation point upstream. But the heat transfer parameter ($(G_\eta)_w$) decreases with the increase of the total enthalpy at the wall (G_w). The reason for this is that when wall temperature is increased, fluid near the wall becomes rarer. This results in reduction in the skin friction at the wall after a certain region of streamwise direction which causes the separation to occur earlier. The effect of decreasing M_∞ results in slight reduction in the values of $(F_\eta)_w$ and $(S_\eta)_w$, and the separation is delayed. Similar effect has been observed by Davis and Walker [28], and more recently by Roy and Nath [17] for non-similar flow

Table 1
Comparison of skin friction and heat transfer parameters with those tabulated by Dewey and Gross [8] for $\beta = 0.5$, $Pr = 1$, $A = \xi = \bar{x} = 0$

G_w	$E \sin^2 \theta$	Present results		In [8]	
		$(F_\eta)_w$	$(S_\eta)_w = (G_\eta)_w$	$(F_\eta)_w$	$(S_\eta)_w = (G_\eta)_w$
0	0.3750	0.6439	0.5071	0.6438	0.5070
0	0.6667	0.7811	0.5330	0.7812	0.5328
0	0.8461	1.0892	0.5829	1.0890	0.5828
0	0.9000	1.3652	0.6213	1.3650	0.6211
0.5	0.3750	0.9169	0.5411	0.9167	0.5410
0.5	0.6667	1.2483	0.5835	1.2480	0.5833
0.5	0.8451	1.9663	0.6578	1.9660	0.6577
0.5	0.9000	2.6004	0.7115	2.6000	0.7113

over two-dimensional and axisymmetric bodies. It has also been found that the yaw angle θ has little effect on the point of separation. Hence, it is not presented here.

3.2. Case II: non-uniform total enthalpy at the wall

The effect of non-uniform total enthalpy at the wall (i.e., the effect of cooling or heating of the wall in a slot) on the skin friction and heat transfer parameters ($(F_\eta)_w$, $(S_\eta)_w$, $(G_\eta)_w$) is shown in Fig. 7. The heat transfer parameter $(G_\eta)_w$ increases due to wall cooling in a slot (Fig. 7), but decreases when there is wall heating in a slot (not shown to reduce the number of figures). The variation of the heat transfer parameter $(G_\eta)_w$ is strongly affected by the variation of the total enthalpy at the wall whereas the skin friction parameters $(F_\eta)_w$ and $(S_\eta)_w$ are affected very little by it (indistinguishable in this scale in Fig. 7) because the heat transfer parameter is more sensitive to the change in the total enthalpy at the wall than skin friction.

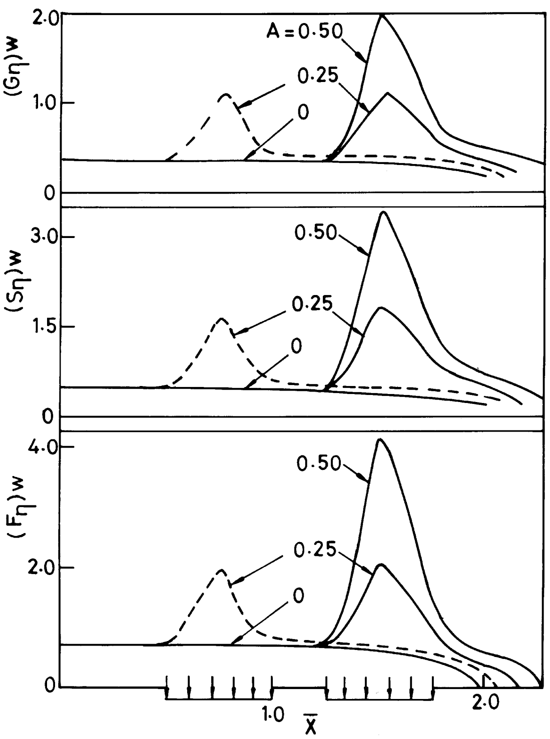

Fig. 4. Effects of suction ($A > 0$) and slot location ($\bar{x}_0$) on skin friction and heat transfer parameters for $M_\infty = 0.4$, $G_w = 0.2$, $w^* = 2\pi$: ———— slot location at $\bar{x}_0 = 1.25$; — — — — slot location at $\bar{x}_0 = 0.50$.

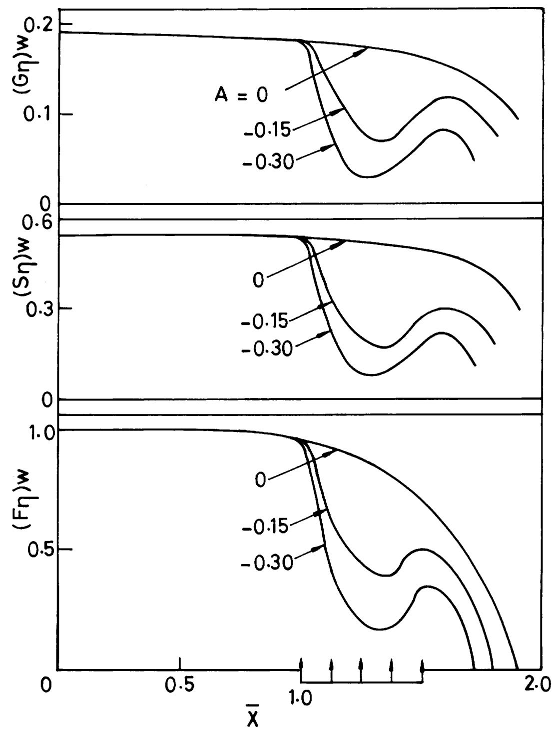

Fig. 5. Effect of injection ($A < 0$) on skin friction and heat transfer parameters for $M_\infty = 0.4$, $G_w = 0.6$, $\bar{x}_0 = 1.0$ and $w^* = 2\pi$.

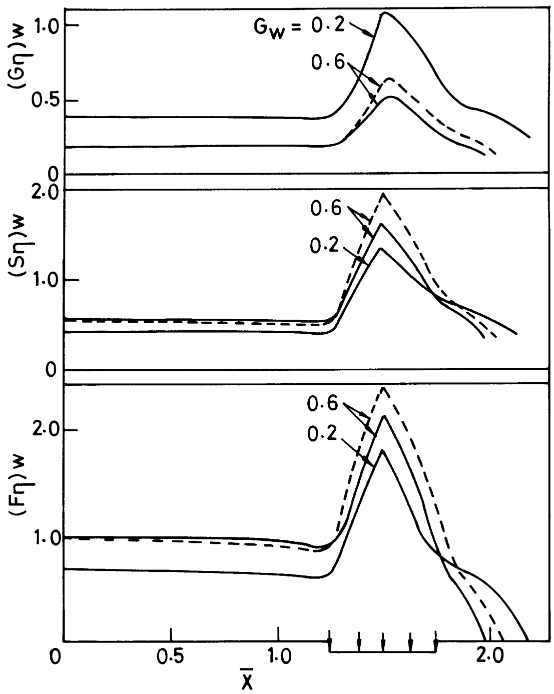

Fig. 6. Effects of Mach number (M_∞) and total enthalpy at wall (G_w) on skin friction and heat transfer parameters for $A = 0.25$, $\bar{x}_0 = 1.25$ and $w^* = 2\pi$: ———— $M_\infty = 0.4$; — — — — $M_\infty = 0.2$.

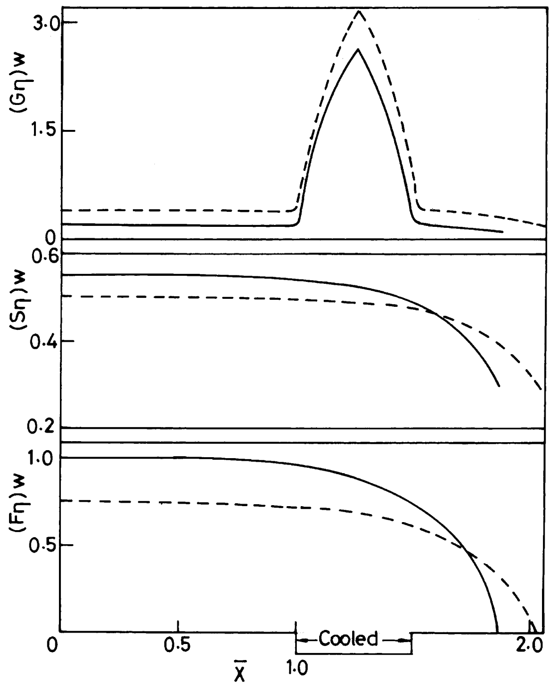

Fig. 7. Effect of wall cooling (in a slot) on skin friction and heat transfer parameters for $A = 0, M_\infty = 0.4$, $\varepsilon = -0.05$, $\bar{x}_0 = 1.0$ and $w^* = 2\pi$: ———— $G_w = 0.6$; — — — $G_w = 0.2$.

4. Conclusions

The results indicate that the separation can be delayed by non-uniform slot suction and also by moving the slot downstream but the non-uniform slot injection has the reverse effect. The increase of total enthalpy at the wall causes the separation to occur earlier while cooling delays it. The increase of Mach number shifts the point of separation upstream due to the adverse pressure gradient. The non-uniform total enthalpy at the wall (i.e., the cooling or heating of the wall in a slot) has very little effect on skin friction and thus on the point of separation. Also the yaw angle has very little effect on the location of the point of separation.

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